An Almost Classical Logic for Logic Programming and Nonmonotonic Reasoning

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1. In a Nutshell

Two levels of modelling are often needed:

- Specifications (e.g. database tuples, logic program clauses)
- Requirements (e.g. database integrity constraints)

Requirements are often expressed as “denials”, e.g.

$$false \leftarrow p(X) \wedge \neg q(X)$$

suggesting that, in a convenient paraconsistent logic, requirements might be expressed using double negation.

Such a logic, N^4, is defined. N^4 is “almost classical”. N^4 turns out to be convenient for nonmonotonic reasoning, too.

2. The Logic N^4

Standard first-order syntax with $\top$ (verum), $\bot$ (falsum), and:

- $(F \rightarrow G) := (\neg F \vee G)$
- $(F \leftrightarrow G) := ((\neg F \vee G) \wedge (\neg G \vee F))$

N^4 positive literal:

- atom
- doubly negated atom

N^4 negative literal:

- negated atom
- threefold negated atom

2. The Logic N^4 (cont'd)

N^4 interpretations are defined very much like classical logic interpretations.

A N^4 interpretation assigns relations to

- predicate symbols
- doubly negated predicate symbols

such that $val(p) \subseteq val(\neg^2 p)$

The truth value of a formula in an N^4 interpretation is defined recursively in terms of the truth value of its subformulas, i.e. compositionally.

2. The Logic N^4 (cont'd)

Possible valuations of p, $\neg p$, $\neg^2 p$, and $\neg^3 p$

- in N^4 interpretations:

p	$\neg p$	$\neg^2 p$	$\neg^3 p$
true	**false**	**true**	**false**
false	**true**	**true**	**false**
false	**true**	**false**	**true**

- in N^4 interpretations satisfying $(\neg p \rightarrow p)$:

p	$\neg p$	$\neg^2 p$	$\neg^3 p$
true	**false**	**true**	**false**
false	**true**	**true**	**false**

2. The Logic N^4 (cont'd)

Properties (hint: read $\neg^2 F$ as "required F"):

- $F \models_{\mathrm{N}^4} \neg^2 F$ (but $\neg^2 F \not\models_{\mathrm{N}^4} F$)
- $\neg F \models_{\mathrm{N}^4} \neg^3 F$ (but $\neg^3 F \not\models_{\mathrm{N}^4} \neg F$)
- Fourfold negation reduction: $\neg^4 F \equiv_{\mathrm{N}^4} \neg^2 F$
- Laws of excluded middle:
  $(F \vee \neg F) \equiv_{\mathrm{N}^4} (\neg F \vee \neg^2 F) \equiv_{\mathrm{N}^4} (\neg^2 F \vee \neg^3 F) \equiv_{\mathrm{N}^4} \top$
- Laws of excluded contradiction:
  $(\neg^2 F \wedge \neg^3 F) \equiv_{\mathrm{N}^4} (F \wedge \neg^3 F) \equiv_{\mathrm{N}^4} \bot$

3. N^4 Herbrand Interpretations

Classical definitions naturally extend to N^4:

- N^4 Herbrand base: Set of all ground positive N^4 literals (i.e. ground atoms and doubly negated atoms)
- A N^4 Herbrand Interpretation $\mathcal{H}$ is characterized by the set M of ground positive N^4 literals it satisfies: $\mathcal{H} = \mathcal{H}(M)$
- Every closed subset M of the N^4 Herbrand base (i.e. if A atom and $A \in M$, then $\neg^2 A \in M$) characterizes a N^4 Herbrand Interpretation $\mathcal{H}(M)$
- Order on N^4 Herbrand Interpretations: $\mathcal{H}(M_1) \preceq \mathcal{H}(M_2)$ iff $M_1 \subseteq M_2$
- Intersection of N^4 Herbrand Interpretations and minimal N^4 Herbrand models

Property: $\mathcal{H}(\bigcap_{i \in I} M_i) = \bigcap_{i \in I} \mathcal{H}(M_i)$

4. Nonmonotonic Reasoning

A characterization of classical minimal models extends to N^4:

Let M be a closed subset of the N4 Herbrand base (i.e. if A atom and $A \in M$, then $\neg^2 A \in M$)

Let $\tilde{M} = \{\neg L \mid L$ in the N^4 Herbrand base and $L \notin M\}$

et $\mathcal{S}$ be a set of formulas

$\mathcal{H}(M)$ is a minimal N^4 Herbrand model of $\mathcal{S}$ iff

- $\mathcal{H}(M) \models_{\mathrm{N}^4} \mathcal{S}$
- For all $L \in M$ $\mathcal{S} \cup \tilde{M} \models_{\mathrm{N}^4} L$

4. Nonmon. Reasoning (cont'd)

Minimal N^4 model of $p \leftarrow \neg p \equiv_{\mathrm{N}^4} (\neg^2 p \vee p)$:

p	$\neg p$	$\neg^2 p$
false	**false**	**true**

Minimal N^4 models of $\{b \leftarrow \neg a \; ; \; a \leftarrow \neg b\} \equiv_{\mathrm{N}^4} \{(b \vee \neg^2 a), (a \vee \neg^2 b)\}$:

a	$\neg a$	$\neg^2 a$	b	$\neg b$	$\neg^2 b$
true	**false**	**true**	**false**	**true**	**false**
false	**false**	**true**	**false**	**false**	**true**
false	**true**	**false**	**true**	**false**	**true**

Property: A N^4 Herbrand model of a normal logic program is stable iff it is complete (i.e. classical) and minimal.

5. N^4 Intuitive Meaning

- A N^4 model of $\mathcal{S}$ in which both $\neg F$ and $\neg^2 F$ are true can be seen as a witness of incorrectness of $\mathcal{S}$, i.e.
  - incorrectness of the requirements, and/or
  - incorrectness of the implementation

- N^4 double negation can be seen as an epistemic modality:
  - $\neg^2 F$ read as "required F" (and $\neg^3 F$ read as "required $\neg F$" or "not required F")
  - this reading fits well with "negation as failure"

6. Conclusion

N^4: A logic for requirement modeling and nonmonotonic reasoning

- N^4 implication is material (i.e. $(F \rightarrow G) \equiv_{\mathrm{N}^4} (\neg F \vee G)$)
- N^4 semantics is compositional
- N^4 naturally extends the Herbrand Model Theory of (negation-free) logic programs to normal logic programs
- N^4 naturally extends the stable model semantics of normal logic programs: Every normal logic program has a minimal N^4 Herbrand Model which is a stable model if it is a classical model