An Almost Classical Logic for Logic Programming and Nonmonotonic Reasoning

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Summary

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6. Conclusion (p. 12)
1. In a Nutshell

Two levels of modelling are often needed:

- Specifications (e.g. database tuples, logic program clauses)
- Requirements (e.g. database integrity constraints)

Requirements are often expressed as “denials”, e.g.

\[ false \leftarrow p(X) \land \neg q(X) \]

suggesting that, in a convenient paraconsistent logic, requirements might be expressed using double negation.

Such a logic, $N^4$, is defined. $N^4$ is “almost classical”. $N^4$ turns out to be convenient for nonmonotonic reasoning, too.
2. The Logic $N^4$

Standard first-order syntax with $\top$ (verum), $\bot$ (falsum), and:

- $(F \rightarrow G) := (\neg F \lor G)$
- $(F \leftrightarrow G) := ((\neg F \lor G) \land (\neg G \lor F))$

$N^4$ positive literal:

- atom
- doubly negated atom

$N^4$ negative literal:

- negated atom
- threefold negated atom
2. The Logic $N^4$ (cont’d)

$N^4$ interpretations are defined very much like classical logic interpretations.

A $N^4$ interpretation assigns relations to

- predicate symbols
- doubly negated predicate symbols

such that $val(p) \subseteq val(\neg^2 p)$

The truth value of a formula in an $N^4$ interpretation is defined recursively in terms of the truth value of its subformulas, i.e. compositionally.
2. The Logic $\mathbb{N}^4$ (cont’d)

Possible valuations of $p$, $\neg p$, $\neg^2p$, and $\neg^3p$

- in $\mathbb{N}^4$ interpretations:

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- in $\mathbb{N}^4$ interpretations satisfying $(\neg p \rightarrow p)$:

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2. The Logic $N^4$ (cont’d)

Properties (hint: read $\neg^2 F$ as “required $F$”):

- $F \models_{N^4} \neg^2 F$  (but $\neg^2 F \not\models_{N^4} F$)
- $\neg F \models_{N^4} \neg^3 F$  (but $\neg^3 F \not\models_{N^4} \neg F$)
- Fourfold negation reduction:  $\neg^4 F \equiv_{N^4} \neg^2 F$
- Laws of excluded middle:
  $F \lor \neg F \equiv_{N^4} (\neg F \lor \neg^2 F) \equiv_{N^4} (\neg^2 F \lor \neg^3 F) \equiv_{N^4} \top$
- Laws of excluded contradiction:
  $\neg^2 F \land \neg^3 F \equiv_{N^4} (F \land \neg^3 F) \equiv_{N^4} \bot$
3. $N^4$ Herbrand Interpretations

Classical definitions naturally extend to $N^4$:

- $N^4$ Herbrand base: Set of all ground positive $N^4$ literals (i.e. ground atoms and doubly negated atoms)

- A $N^4$ Herbrand Interpretation $\mathcal{H}$ is characterized by the set $M$ of ground positive $N^4$ literals it satisfies: $\mathcal{H} = \mathcal{H}(M)$

- Every closed subset $M$ of the $N^4$ Herbrand base (i.e. if $A$ atom and $A \in M$, then $\neg^2 A \in M$) characterizes a $N^4$ Herbrand Interpretation $\mathcal{H}(M)$

- Order on $N^4$ Herbrand Interpretations: $\mathcal{H}(M_1) \leq \mathcal{H}(M_2)$ iff $M_1 \subseteq M_2$

- Intersection of $N^4$ Herbrand Interpretations and minimal $N^4$ Herbrand models

Property: $\mathcal{H}(\bigcap_{i \in I} M_i) = \bigcap_{i \in I} \mathcal{H}(M_i)$
A characterization of classical minimal models extends to $\mathbb{N}^4$:

Let $M$ be a closed subset of the $\mathbb{N}^4$ Herbrand base (i.e. if $A$ atom and $A \in M$, then $\neg^2 A \in M$)

Let $\tilde{M} = \{ \neg L \mid L \text{ in the } \mathbb{N}^4 \text{ Herbrand base and } L \notin M \}$

Let $S$ be a set of formulas

$\mathcal{H}(M)$ is a minimal $\mathbb{N}^4$ Herbrand model of $S$ iff

- $\mathcal{H}(M) \models_{\mathbb{N}^4} S$
- For all $L \in M \cup \tilde{M}$, $S \cup \tilde{M} \models_{\mathbb{N}^4} L$
4. Nonmon. Reasoning (cont’d)

Minimal \( \mathbb{N}^4 \) model of \( p \leftarrow \neg p \equiv_{\mathbb{N}^4} (\neg^2 p \lor p) \):

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Minimal \( \mathbb{N}^4 \) models of \( \{b \leftarrow \neg a ; a \leftarrow \neg b\} \equiv_{\mathbb{N}^4} \{(b \lor \neg^2 a), (a \lor \neg^2 b)\} \):

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Property: A \( \mathbb{N}^4 \) Herbrand model of a normal logic program is stable iff it is complete (i.e. classical) and minimal.
5. $N^4$ Intuitive Meaning

- A $N^4$ model of $S$ in which both $\neg F$ and $\neg^2 F$ are true can be seen as a witness of incorrectness of $S$, i.e.
  - incorrectness of the requirements, and/or
  - incorrectness of the implementation

- $N^4$ double negation can be seen as an epistemic modality:
  - $\neg^2 F$ read as “required $F$” (and $\neg^3 F$ read as “required $\neg F$” or “not required $F$”)
  - this reading fits well with “negation as failure”
N⁴: A logic for requirement modeling and nonmonotonic reasoning

- N⁴ implication is material (i.e. \((F \rightarrow G) \equiv_{N^4} (\neg F \lor G)\))
- N⁴ semantics is compositional
- N⁴ naturally extends the Herbrand Model Theory of (negation-free) logic programs to normal logic programs
- N⁴ naturally extends the stable model semantics of normal logic programs: Every normal logic program has a minimal N⁴ Herbrand Model which is a stable model if it is a classical model